Maxwell's

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$\nabla \times E = 0$

Q1. a) $\oint E\, ds = \frac{Q}{\varepsilon_0} = 0$, pois a carga está em condutor perfeito na superfície.

b) $E = \frac{\sigma A}{\varepsilon A} = \boxed{\frac{\sigma}{\varepsilon_0}\hat{n}}$

c) não há campo elétrico tangente à superfície $= 0$ (estático); se existisse iria se
se tiver componente paralela vai ocorrer acelerando as cargas e não será eletrostático   autocorreção

Q2. a) $\vec{v}(x,t) = Ae^{i(\vec{k}\vec{x} - \omega t)}$

$$\boxed{\nabla \cdot v = \frac{\partial v_x}{\partial x} = ik\, v(x,t)}$$

$v_x = 0$

$$ik \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ ik & ik & ik \\ v_x & v_y & v_z \end{vmatrix} = ik v_x \hat{y} - ik v_x \hat{z}$$

$$ik(v_z - v_y)\hat{x} + ik(-v_z)\hat{y} + ik(v_y)\hat{z} = \boxed{-ik v_y \hat{y} + ik v \hat{z}}$$

b) $$\nabla \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} \Rightarrow -\frac{\partial v}{\partial x}\hat{y} + \frac{\partial v}{\partial x}\hat{z} = \boxed{-ik\, v(x,t)\hat{y} + ik\, v(x,t)\hat{z}}$$

c) direção propagação x

$\nabla \cdot E = \nabla \cdot B = 0$ (vácuo)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \Rightarrow \frac{\partial E_x}{\partial x} = \frac{\partial B_x}{\partial x} = 0$$

não varia em x, direção de propagação: onda transversal.

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

$$\frac{i k \times E}{w} = B$$

$$E \cdot B = \frac{\overbrace{E \cdot i k \times E}^{=0}}{w} = 0$$ perpendicular

Q3.

um electrão

a) $L = \hbar n = m v R$ $\qquad F_{el} = m a_c$ $\qquad m^2 v^2 r^2 = L^2 = \hbar^2 n^2$

$$\frac{Z e^2}{4\pi\varepsilon_0 r^2} = \frac{m v^2}{r} \Rightarrow \frac{r_n \cdot Z e^2 \cdot m}{4\pi\varepsilon_0} = (n\hbar)^2$$

$$p^2 = \frac{Z e^2 \cdot m}{4\pi\varepsilon_0 r}$$

$$\boxed{r_n = \frac{4\pi\varepsilon_0 (n\hbar)^2}{Z m e^2}}$$

b) $$E = \frac{p^2}{2m} + \frac{Z e^2}{4\pi\varepsilon_0 r_n} = \frac{Z e^2}{2\, 4\pi\varepsilon_0 r_n} - \frac{Z e^2}{4\pi\varepsilon_0 r_n} = -\frac{Z e^2}{2 \cdot 4\pi\varepsilon_0 r_n} = -\frac{m (Z e^2)^2}{2 \cdot (4\pi\varepsilon_0 n\hbar)^2} \simeq \frac{-13{,}6 Z^2}{n^2} eV$$

$E_0 = \frac{e^2}{2(4\pi\varepsilon_0) r_n}$

$$\Delta E = E_1 - E_0 = \frac{-13{,}6}{4} + 13{,}6 = \boxed{10{,}2\ eV} \qquad E_n = \frac{-13{,}6}{n^2}$$

$$+\frac{E_0}{4} + E_n = \frac{-3E_0}{4}$$

2

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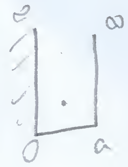
Q3.

c)



$\oint p\,dx = hn$

$E = \frac{p^2}{2m} + \cancel{V}^{0} = \int_0^a \frac{p^2}{2m} dx$

$\downarrow$

$p^2 = 2mE$

$p = \sqrt{2mE} \Rightarrow \int_0^a p\,dx = hn \Rightarrow \sqrt{2mE}\cdot a = hn$

$$\boxed{E_n = \left(\frac{hn}{a}\right)^2 \frac{1}{2m}}$$

d) $E_1 - E_0 = -\frac{3E_0}{4}$

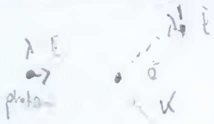
$\left(\frac{2h}{a}\right)^2 \cdot \frac{1}{2m} - \left(\frac{h}{a}\right)^2 \frac{1}{2m} = -\frac{3E_0}{4}$ $\qquad E_0 = -\frac{e^2}{2(4\pi\epsilon_0)r_n}$

$\cancel{3}\left(\frac{h}{a}\right)^2 \frac{1}{\cancel{2}m} = -\frac{\cancel{3}E_0}{\cancel{4}_2} \Rightarrow \left(\frac{h}{a}\right)^2 = -\frac{mE_0}{2} \Rightarrow a = h\left(-\frac{2}{mE_0}\right)^{\frac{1}{2}}$

$$\boxed{a = \left(\frac{4(4\pi\epsilon_0)r_n}{me^2}\right)^{\frac{1}{2}}}$$

$\Delta x = x - x_0 \Rightarrow x_0 = x + \Delta x$

Q4.



$\lambda' - \lambda^0 = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$

$E = \frac{hc}{\lambda}$

$\frac{hc}{\lambda} - \frac{hc}{\lambda'} = K$ $\qquad$ p/ max K → Δλ tem que ser max → $\boxed{\theta = 180^0}$

$\lambda = \frac{hc}{m_0 c^2} = \frac{h}{m_0 c}$ $\qquad$ $m_0 c^2 = E$ $\quad \boxed{\frac{hc}{\lambda}} - \frac{hc}{(\lambda + \Delta\lambda)} = K$ $\qquad \boxed{\Delta\lambda = \frac{2h}{m_0 c}}$

$\Delta\lambda = \frac{2hc}{E} \Rightarrow E - \frac{hc}{\left(\lambda + \frac{2hc}{E}\right)} = K_{max} \Rightarrow E - \frac{hcE}{(\lambda E + 2hc)} = K$

$\frac{\lambda E^2 + 2hcE - hcE}{\lambda E + 2hc} = K$

$\frac{\lambda E^2 + hcE}{\lambda E + 2hc} = K_{max}$?

b) $\Delta\lambda = \frac{h}{m_0 c}\left(1 - \cos(120°)\right) \Rightarrow E = \frac{hc}{(\lambda + \Delta\lambda)}$; $K = \frac{hc}{\lambda} - \frac{hc}{(\lambda + \Delta\lambda)}$

c) 

$\cot\left(\frac{\theta}{2}\right) = \left(1 + \frac{\lambda_c}{\lambda_0}\right)\tan\phi$ a direção correspondente do momento e calcula ângulo ϕ

05.

05. $V \to 2V_0$ W? Q? (V_0, p_0) e c

a) $\boxed{W = 0}$; $\boxed{Q = 0} \Rightarrow \boxed{\Delta U = 0} \Rightarrow \Delta T = 0$, $\boxed{T_i = T_f}$ $\Delta U = n c_v \Delta T$

$U(T, V) \to dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV = 0$

$d(TS) = dT \cdot S + T dS$

$d(U - TS) = -S dT - p dV$ ✗ $p \Delta V = nR\Delta T$

$dT = -\left(\frac{\partial T}{\partial V}\right)_U dV$ $\left(\frac{\partial U}{\partial V}\right)_T =$

$dT = -\frac{\left(\frac{\partial U}{\partial V}\right)_T}{\left(\frac{\partial U}{\partial T}\right)_V} dV$

T =

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b) $dU = T\cancel{dS}^{0} + p\,dV$

$dU = p\,dV$

c) isobárica => pressão constante

$\frac{V}{T} = K$ (constante)

$V = KT \Rightarrow T_i = \frac{V_0}{K}$

$T_f = \frac{2V_0}{K}$

$\Delta T = \frac{V_0}{K}$

$\boxed{W = p_0 \Delta V = p_0 V_0}$

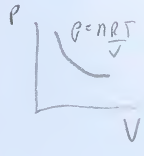
$Q = m \cdot c_p \Delta T = \boxed{\frac{m c_p V_0}{K}}$

$\Delta U = cR\Delta T$

$W = p_0 \Delta V \Rightarrow Q = \Delta U + W = cR\Delta T + p_0 \Delta V$

$$\boxed{Q = cR\frac{V_0}{K} + p_0 V_0}$$

d) isotérmica => T constante



$\Delta T = 0,\ Q \neq 0$

$pV =$ constante

T constante pode sair da integral

$$W_{A \to B} = \int_{V_A}^{V_B} p\,dV = \int_{V_0}^{2V_0} \frac{RT}{V}\,dV = RT\ln(2)$$

$dU = 0 \Rightarrow \Delta U = cR\Delta T = 0$

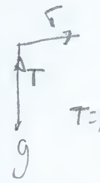
$\boxed{W = Q}$

Q6. A B

m M

$w = vR$

a)



$F_T = ma_c = \frac{mv^2}{r_0} = Mg \Rightarrow m\omega^2 r_0 = Mg$

$T = Mg$

$$\boxed{\omega_0 = \sqrt{\frac{Mg}{mr_0}}}$$

b) $L = \frac{1}{2}(m+M)\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - Mg(r-l)$

r: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \quad \ddot{r}(m+M) - mr\dot{\theta}^2 + Mg = 0$

$$\boxed{\ddot{r} - \frac{mr\dot{\theta}^2}{(m+M)} + \frac{Mg}{(M+m)} = 0}$$

θ:

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \Rightarrow mr^2\ddot{\theta} = 0 \Rightarrow \boxed{\ddot{\theta} = 0}$

c) if $\frac{\partial L}{\partial q} = 0 \Rightarrow p = \frac{\partial L}{\partial \dot{q}}$ conserved

$L = mvr = rp = r \times p$

$\frac{\partial L}{\partial \theta} = 0 = p = \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} \overset{\text{ang mom?}}{=} L$ (momentum conserved!)

$r\dot{\theta} = v$

mvr

$\frac{\partial L}{\partial t} = 0 \Rightarrow$ Energy conserved

d)

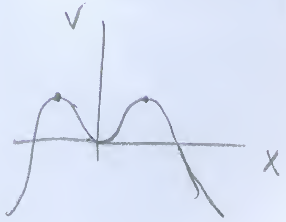
EvP - 2014 ... var

Q1 $V(x) = \frac{1}{2}kx^2 - \frac{k}{4a^2}x^4$

$\left(-1 + \frac{x^2}{a^2}\right) < 0$

a) $F(x) = -\nabla V = -\cancel{kx} + \frac{kx^3}{a^2} \Rightarrow F = 0 = -kx + \frac{kx^3}{a^2}$

$\boxed{x = 0, \pm a}$



$x = 0 \Rightarrow$ estável

$x = \pm a \Rightarrow$ instável.

$\omega^2 = \frac{k}{m}$

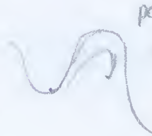
b) $\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=0} = k - \frac{3kx^2}{a^2} = k \Rightarrow m\omega^2 \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

c) $V(a) = \boxed{\frac{1}{4}ka^2}$, $F(a) = 0$ $\quad 0 < v < a\sqrt{\frac{k}{2m}}$ $\quad v > a\sqrt{\frac{k}{2m}}$

$E = \frac{mv^2}{2} \Rightarrow \frac{m a^2 k}{2 \cdot 2m} = \boxed{\frac{a^2 k}{4}}$

partícula presa no poço

partícula super barreira



d) $-kx + \frac{kx^3}{a^2} = m\ddot{x}$ $\Rightarrow \boxed{E = \frac{m\dot{x}^2}{2} + \frac{1}{2}kx^2 - \frac{kx^4}{4a^2}}$

x

$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + \frac{k}{4a^2}x^4$ $\qquad \frac{m\dot{x}^2}{2} - \left(ka - \frac{\sqrt{k}x^2}{2a}\right)^2 + k^2a^2 = E$

$\frac{d}{dt}(m\dot{x}) + kx - \frac{kx^3}{a^2} = 0$ $\Rightarrow \ddot{x} + \omega^2 x - \frac{\omega^2}{a^2}x^3 = 0$

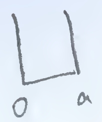
$\frac{\dot{x}^2}{2} = \frac{\omega^2}{a^2}\int \dot{x}(x^3 - ax)\,dx$

$\int \dot{x}\ddot{x} = \int \frac{\omega^2}{a^2}\dot{x}(x^3 - a^2x)$ $\qquad \frac{\dot{x}^2}{2} = \frac{\omega^2}{a^2}\left(\frac{x^4}{4} - \frac{ax^2}{2}\right)$

$\ddot{x} + \frac{k}{m}x - \frac{kx^3}{a^2m} = 0$

(1)

Q8.



$\hat{p} = -i\hbar \frac{\partial}{\partial x}$

a) $\hat{H}\psi(x) = E\psi(x)$

$\frac{\hat{p}^2}{2m}\psi(x) = E\psi(x)$

$\Rightarrow -\frac{\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} - E\psi(x) = 0$

$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar}\psi(x) = 0 \qquad k^2 = \frac{2mE}{\hbar}$

$\Delta = -4k^2 \Rightarrow r = \pm ik$

$\psi(x) = Ae^{-kx} + Be^{kx} \Rightarrow \psi(x) = A\sin(kx)$

$x = 0 \Rightarrow \psi = 0 \;\&\; x = a$

$\sin(a \cdot k) = 0 \Rightarrow k = \frac{n\pi}{a} \Rightarrow$

$\Rightarrow \psi(x) = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi}{a}x\right)$

$\int_0^a |\psi(x)|^2 dx = 1 \Rightarrow \frac{A^2}{2}a = 1 \Rightarrow A = \sqrt{\frac{2}{a}}$

$E = \frac{\hbar k^2}{2m} = \frac{\hbar n^2\pi^2}{2a^2 m}$

$n = 2 \Rightarrow E = \frac{2\hbar\pi^2}{a^2 m}$

b) $\int_0^a A^2\left[\sin^2\left(\frac{\pi x}{a}\right) + 9\sin^2\left(\frac{2\pi x}{a}\right)\right]dx = 1$

$\frac{A^2}{2}\int_0^a \left[10 + \cos\left(\frac{2\pi x}{a}\right) + 9\cos\left(\frac{4\pi x}{a}\right)\right]dx = 1$

$\frac{A^2}{2}\left[10a + \left(-\sin\left(\frac{2\pi x}{a}\right)\cdot\frac{a}{2\pi}\right)\Big|_0^a + \left(-9\sin\left(\frac{4\pi x}{a}\right)\cdot\frac{a}{4\pi}\right)\Big|_0^a\right] = 1$

$A = \sqrt{\frac{1}{5a}}$

$\psi_2(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{2\pi}{a}x\right)$

$P(\psi_2) = \frac{1}{5a}\cdot 9 = \boxed{\frac{9}{5a}}$

c) $\psi_0 = \sqrt{\frac{1}{5a}}\sin\left(\frac{\pi x}{a}\right) \Rightarrow$

$\sin(x) = \frac{e^{+ix} - e^{-ix}}{2i}$

$\psi(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_0^a dx\, e^{-\frac{i}{\hbar}px}\psi(x)$

$\psi(p) = \frac{1}{\sqrt{10\pi\hbar a}}\int_0^a \frac{1}{2i}\left[e^{ix\left(-\frac{p}{\hbar}+\frac{\pi}{a}\right)} - e^{-ix\left(+\frac{p}{\hbar}+\frac{\pi}{a}\right)}\right]dx$

08. c) EUF 2014-1sem 10/03/2016

$$\Psi(p) = \frac{-i}{2\sqrt{10\pi\hbar a}} \left[-i\frac{e^{ix\left(-\frac{p}{\hbar}+\frac{\pi}{a}\right)}}{\left(-\frac{p}{\hbar}+\frac{\pi}{a}\right)} - i\frac{e^{-ix\left(\frac{p}{\hbar}+\frac{\pi}{a}\right)}}{\left(\frac{p}{\hbar}+\frac{\pi}{a}\right)} \right]_0^a$$

$$\Psi(p) = -\frac{1}{2\sqrt{10\pi\hbar a}} \left[\frac{e^{-\frac{iap}{\hbar}} \cdot e^{i\pi}}{\left(-\frac{p}{\hbar}+\frac{\pi}{a}\right)} - \frac{1}{\left(-\frac{p}{\hbar}+\frac{\pi}{a}\right)} + \frac{e^{-\frac{iap}{\hbar}} \cdot e^{-i\pi} - 1}{\left(\frac{p}{\hbar}+\frac{\pi}{a}\right)} \right]$$

$$\Psi(p) = \frac{-e^{-\frac{iap}{\hbar}}}{2\sqrt{10\pi\hbar a}} \left[\frac{e^{i\pi} - 1}{\left(-\frac{p}{\hbar}+\frac{\pi}{a}\right)} + \frac{e^{-i\pi} - 1}{\left(\frac{p}{\hbar}+\frac{\pi}{a}\right)} \right]$$

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d) $\Psi(x) = e^{ikx}$ $\quad -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + 0 = E\Psi$

$$k^2 = \frac{2mE}{\hbar^2} \Rightarrow \boxed{E = \frac{\hbar^2 k^2}{2m}} = \hbar w \qquad E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

09. $\vec{M} = \gamma\vec{S}$; $|+\rangle, |-\rangle, \hat{S}_z$; $\hat{S}_z|+\rangle = \frac{\hbar}{2}|+\rangle$, $\hat{S}_z|-\rangle = -\frac{\hbar}{2}|-\rangle$

$\vec{B} = B\hat{y}$, $\hat{H} = -\vec{M}\cdot\vec{B} = -\gamma B\hat{S}_y$; $|\Psi(0)\rangle = |+\rangle$

a) $\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \Rightarrow \hat{S}_y|+\rangle = i\frac{\hbar}{2}|-\rangle$, $\hat{S}_y|-\rangle = -i\frac{\hbar}{2}|+\rangle$

$\hat{S}_z = \begin{pmatrix} \frac{\hbar}{2} & 0 \\ 0 & -\frac{\hbar}{2} \end{pmatrix}$ $\quad \langle +|\hat{S}_z|+\rangle = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2}$

b) $\frac{\hbar}{2}\begin{vmatrix} -\lambda & -i\frac{\hbar}{2} \\ i\frac{\hbar}{2} & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \frac{\hbar^2}{4} = 0$; $\lambda = \pm\frac{\hbar}{2}$

$\lambda = \frac{\hbar}{2}$ $\Rightarrow \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} a \\ b \end{bmatrix}$, $\begin{bmatrix} 1 \\ i \end{bmatrix} \Rightarrow |+\rangle_y = A\left[|+\rangle + i|-\rangle\right]$

$-ib = \frac{\hbar}{2}a$; $ia = \frac{\hbar}{2}b$; $a = \frac{\hbar^2}{4}$; $b = i\frac{\hbar}{2}a$

$2A^2 = 1$; $A = \frac{1}{\sqrt{2}}$; $\boxed{\frac{1}{\sqrt{2}}|+\rangle \pm \frac{i}{\sqrt{2}}|-\rangle}$

(5)

c) $|\Psi(0)\rangle = |+\rangle$

$$\hat{H} = -\gamma B \hat{S}_y = -\gamma B \cdot \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = -\gamma B\left(+\frac{i\hbar}{2}|-\rangle - \frac{i\hbar}{2}|+\rangle\right)$$

$$\omega = \gamma B \Rightarrow \boxed{\hat{H} = -\frac{\omega\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}}$$

$$|\Psi(t)\rangle = a(t)|+\rangle + b(t)|-\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = H\Psi(x,t)$$

$$i\hbar \frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = -\frac{\omega\hbar}{2}\begin{pmatrix} 0 & -i \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\cancel{i\hbar}\,\dot{a}(t) = \cancel{+i}\frac{\omega\cancel{\hbar}}{2} = +\frac{\omega}{2}a(t) \quad ; \quad \dot{b}(t) = -\frac{\omega\cancel{\hbar}}{2}b(t)$$

$$\ln|a(t)| = \frac{\omega t}{2} \Rightarrow a(t) = e^{\frac{\omega t}{2}}\cdot a(0) \quad ; \quad b(t) = b(0)\cdot e^{-\frac{\omega t}{2}}$$

$$\boxed{|\Psi(t)\rangle = a(0)\cdot e^{\frac{\omega t}{2}}|+\rangle + b(0)\,e^{-\frac{\omega t}{2}}|-\rangle}$$

$$|\Psi(0)\rangle = a(0)|+\rangle + b(0)|-\rangle \Rightarrow b(0) = 0,\ a(0) = 1$$

$$\boxed{|\Psi(t)\rangle = e^{\frac{\omega t}{2}}|+\rangle}$$

d) $\langle S_x\rangle = \langle\Psi(t)|S_x|\Psi(t)\rangle$

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} e^{\frac{\omega t}{2}} & 0 \end{pmatrix}\cdot\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} e^{\frac{\omega t}{2}} \\ 0 \end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix} e^{\frac{\omega t}{2}} & 0 \end{pmatrix}\begin{pmatrix} 0 \\ e^{\frac{\omega t}{2}} \end{pmatrix} = 0$$

EUF 2014 - 1sem. 11/03

d)

$$\langle S_y \rangle = \langle \psi(t) | S_y | \psi(t) \rangle = \left(e^{\frac{\omega t}{2}} \; 0 \right) \frac{-i\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} e^{\frac{\omega t}{2}} \\ 0 \end{pmatrix}$$

$$\hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \frac{-i\hbar}{2} \cdot \left(e^{\frac{\omega t}{2}} \; 0 \right) \begin{pmatrix} 0 \\ \; \end{pmatrix} = 0$$

$$\langle \hat{S}_t \rangle = \left(e^{\frac{\omega t}{2}} \; 0 \right) \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} e^{+\frac{\omega t}{2}} \\ 0 \end{pmatrix} = \left(e^{\frac{\omega t}{2}} \; 0 \right) \frac{\hbar}{2} \begin{pmatrix} e^{+\frac{\omega t}{2}} \\ 0 \end{pmatrix}$$

$$\hat{S}_t = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad ? \qquad \boxed{= \frac{\hbar}{2} \cdot e^{\omega t}}$$

Q12

a) $$3 = \sum_n e^{-\beta E_n} = e^{+\beta \mu H} + 2 \Rightarrow \ln\left(e^{\beta \mu H} + 2\right)$$

$$Z = 3^N = \left(e^{\beta \mu H} + 2\right)^N$$

$$F = -k_B T \ln Z = \frac{-N}{\beta} \cdot \ln\left(e^{\beta \mu H} + 2\right) = -\frac{N}{\beta}\left[\ln\left[e^{+\beta\mu H}\left(1 + 2 e^{-\beta \mu H}\right)\right]\right]$$

$$= -\frac{N}{\beta} \cdot \left[\beta \mu H + \ln\left(1 + 2 e^{-\beta\mu H}\right)\right] \rightarrow \text{sistema}$$

b) $$U = -\frac{\partial}{\partial \beta} \ln 3 = \frac{1}{\left(e^{\beta \mu H} + 2\right)} \cdot (\mu H) e^{\beta \mu H}$$

$$f = -k_B T \ln 3 = -\frac{1}{\beta} \ln\left(e^{\beta \mu H} + 2\right) \qquad s = \frac{f - U}{T} =$$

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c) $\langle \mu_n \rangle = m = \mu \sum_{i=1}^{N} \bar{n}_i \Rightarrow M = -\frac{\partial A}{\partial H} = -\frac{1}{\cancel{\beta}} \cdot \frac{1}{(e^{\beta \mu H} + 2)} \cdot \beta N = \boxed{\frac{N}{e^{\beta \mu H} + 2}}$ ✗

d) $\chi_T = \left(\frac{\partial m_y}{\partial A} \right)_T$

$\langle \mu_n \rangle = m_x \hat{x} + m_y \hat{y} = \mu \hat{y}$

$m_x = \mu \hat{x} + (-\mu \hat{x}) = 0$

$m_y = \mu \cdot P(\mu_y) \hat{y} =$